FD-TLM Modeling of Josephson Junction Circuits

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***Abstract*—Josephson junction circuits are capable of operating at high frequencies where a full-wave field simulation is needed to completely analyze the circuits. The FD-TLM time-domain full-wave electromagnetic field calculation method is extended to modeling high-speed Josephson junction (JJ) circuits by the incorporation of a model for the JJ. The JJ model implementation is described and is validated by excellent agreement between FD-TLM and conventional circuit simulations of the transient response of a JJ circuit having short interconnections.**

I. Introduction

JOSEPHSON junction (JJ) logic circuits provide ultra-high-speed computation with extremely low power dissipation [1]. Because of the high frequencies present in JJ circuits, the dimensions of the interconnections and packaging may be a significant fraction of a wavelength and thus a full wave field analysis would be necessary. The Finite-Difference Transmission Line Matrix (FD-TLM) method is well suited for time-domain simultaneous full-wave field and device analysis of high-speed circuits, and has been used to simulate digital logic circuits using a large-signal nonlinear GaAs MESFET model [2], [3]. Related to the FD-TLM method are the TLM and FDTD methods in which two-terminal semiconductor diodes have been incorporated [4], [5].

In this letter, incorporation of a JJ model in the FD-TLM method is described. An FD-TLM JJ circuit simulation is performed with short interconnections so that comparison of results with a conventional circuit analysis technique can be used to validate the FD-TLM JJ model incorporation. Excellent agreement is obtained between FD-TLM and conventional circuit simulations for the transient analysis of a JJ test circuit.

II. Incorporation of Josephson Junction

The following JJ equations in [6] are implemented in the FD-TLM method

$$I = I_o \sin(\phi) + G(V) + C_j \frac{dV}{dt} \tag{1}$$

$$P_o V = \frac{d\phi}{dt} \tag{2}$$

$$G(V) = G_1 V + (I_1 + G_2|V|)\left\{\frac{1}{1+\exp[(V_s - V)/V_t]} - \frac{1}{1+\exp[(V_s + V)/V_t]}\right\} \tag{3}$$

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where I is the total JJ current, V is the voltage across the JJ, I_o is the JJ critical current, ϕ is the phase difference between the wave functions on each side of the junction, C_j is the junction capacitance, $G(V)$ is the sum of the tunneling and leakage currents, P_o is the plasma oscillation frequency, and t is time. The parameters in (3) are discussed in [6]. In the FD-TLM method, the JJ is implemented as a lumped-circuit element at a shunt (electric field) node, for example an E_z node. Voltage V is calculated as $V = -V_z = -E_z \Delta z$, where V_z is the voltage across the E_z shunt node, E_z is the electric field, and Δz is the size of the grid in the z-direction surrounding the E_z node [3]. The JJ capacitance C_j, which passes the JJ displacement current, is implemented as a lumped capacitance at this node following the procedure in [3]. The current component $G(V)$ is implemented as a time- and voltage-varying conductivity at the E_z node, analogous to the diode conduction current implemented in [3]. Total current I is defined as positive when flowing in the negative z-direction.

The JJ superconducting current $I_S = I_o \sin(\phi)$ in (1) is incorporated in the FD-TLM method following a procedure analogous to the implementation of a resonant tunneling diode in the FDTD method [5]. The FD-TLM equation for the E_z electric field node (11) in [3] is modified by adding the time-average superconducting current $I_{S\mathrm{ave}} = [I_S(V^{n+1}) + I_S(V^n)]/2$ to the right side, where V^n is the JJ voltage at time $n\tau$, V^{n+1} is the new JJ voltage at time $(n+1)\tau$, and τ is the FD-TLM time step. Current $I_S(V^n)$ is calculated as $I_o \sin(\phi^n)$, where ϕ^n is the phase at time $n\tau$ and is obtained by integrating (2) as the sum of the product $P_o V^n \tau$ at each time step. At the start of the simulation the phase ϕ is initially zero. The new superconducting current is found as

$$I_S(V^{n+1}) \approx I_S(V^n) + \frac{dI_S(V = V^n)}{dV}(V^{n+1} - V^n) \tag{4}$$

using a truncated Taylor series. The derivative in (4) is

$$\frac{dI_S(V = V^n)}{dV} = I_o \cos(\phi^n) P_o V^n \frac{\tau}{V^{n+1} - V^n}. \tag{5}$$

Incorporating the time-average JJ superconducting current in (11) of [3] yields the new FD-TLM equation for updating the E_z electric field node voltage

$$\begin{aligned} V_z^{n+1} = {} & \left[1 - \frac{2Y_{Lzn}}{K} - \frac{Z_0}{K} I_o \cos(\phi^n) P_o \tau\right] V_z^n \\ & + \frac{2Z_0}{K}\left[I_{x,j-1}^{n+(1/2)} - I_x^{n+(1/2)} + I_y^{n+(1/2)} - I_{y,i-1}^{n+(1/2)} \right. \\ & \left. + I_o \sin(\phi^n)\right] \end{aligned} \tag{6}$$

where V_z^{n+1} and V_z^n are the shunt node voltages and the subscripted I are the currents flowing in neighboring series (magnetic field) nodes. The superscripts indicate the time step.

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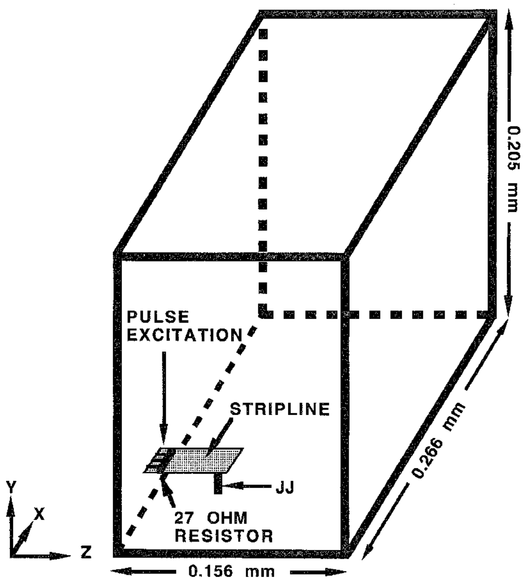

Fig. 1. Josephson junction circuit modeled using the FD-TLM method.

The parameters in (6) are discussed in [3]. Josephson junctions oriented along the x or y axis are implemented similarly at an E_x or E_y field node, respectively.

III. Simulation Results

Fig. 1 shows the JJ test structure simulated in a perfectly-conducting box using the FD-TLM method. The E_z electric field nodes between the left box wall and the resistor are excited to produce a voltage pulse with zero initial value, 5 mV peak value for 20 ps, and 3 ps rise and fall times. A 10-μm-long, 5-μm-wide, infinitely-thin, perfectly-conducting stripline connects the resistor to the top of the JJ, which is implemented at an E_y node. The stripline is short so that it will contribute little parasitic inductance and capacitance. The return path for the JJ current is through the box bottom and side wall. Surrounding the JJ are $1 \times 1 \times 1$ $(\mu\text{m})^3$ cubic FD-TLM grid cells which are graded into larger cells away from the circuit. The *fdtgraph* graphical user interface is used to create the geometrical description of the structure for FD-TLM simulation [7].

Fig. 2 shows the JJ voltage response calculated by FD-TLM and conventional circuit simulations. The voltages are in close agreement and thus the lines overlap in Fig. 2. The JJ current is increased quickly in 3 ps, causing the JJ to go into the high voltage state (HVS) [6]. There is a small-amplitude, high-frequency oscillation when the JJ voltage is 2 mV. After the pulse excitation turns off, the oscillation grows and then decays in amplitude while decreasing in frequency as the JJ progresses toward the zero voltage state (ZVS) [6]. In Fig. 3, the coincidence of the traces indicates excellent agreement between the JJ current from the FD-TLM and conventional circuit simulations, validating the implementation of the JJ model in the FD-TLM method. In the FD-TLM simulation, the current is calculated by integrating the transverse magnetic field around a loop surrounding the stripline, and the voltage is calculated from the E_y node representing the JJ. The conventional circuit analysis uses Kirchhoff's voltage law

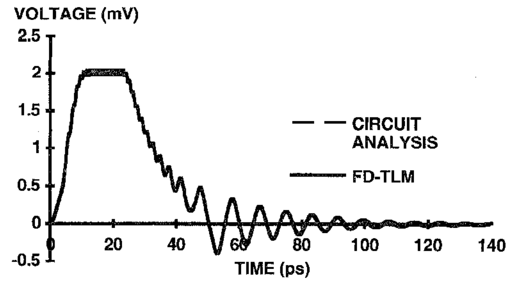

Fig. 2. Josephson junction voltage from FD-TLM and conventional circuit simulations.

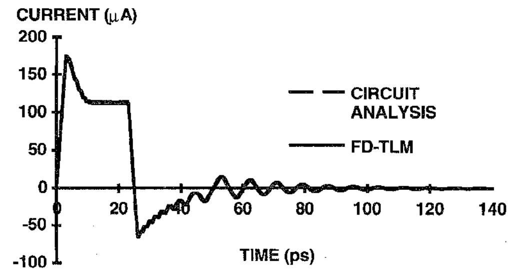

Fig. 3. Josephson junction current from Fd-TLM and conventional circuit simulations.

and solves (1)–(3) with a fifth-order Runge-Kutta numerical technique [8]. In both simulations the JJ parameters are $I_o = 0.1\,\text{mA}, C_j = 0.5\,\text{pF}, V_s = 2\,\text{mV}, V_t = 0.1\,\text{mV}, G_1 = 4\,\text{m℧}, G_2 = 0.1\,\text{℧}$, and $I_1 = 0\,\text{A}$.

IV. Conclusion

A model for the Josephson junction has been sucessfully incorporated in the FD-TLM electromagnetic field simulation method. Interconnection length in the FD-TLM simulation of a JJ test circuit was intentionally minimized so that a conventional circuit simulation technique could validate the JJ model implementation in the FD-TLM method. Excellent agreement between the two numerical techniques was obtained. Research is underway in modeling more complex JJ circuits.

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